THE MEASUREMENT OF TRANSISTOR SMALL Y PARAMETERS

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ABSTRACT

Definitions of small signal y and h parameters are reviewed together with conversion formulas for y parameters in the three basic transistor configurations. The behavior of transistor parameters as a function of frequency and operating point is then analyzed starting from the behavior of some other well-known basic parameters.

Finally, experimental procedures required for the measurement of y parameters from 50 kc to 100 mc are described and some typical results are shown. Particular attention is paid to the practical points involved.

1.0 INTRODUCTION

The importance of characterizing the gain and impedance properties of transistors by means of small signal parameters is obvious, particularly at high frequencies. There are several methods for such characterization and each method is capable of giving some particular information about the performance of the device. For example, equivalent circuit representations of various forms are sometimes used because they give an insight into the working mechanism of the device, each circuit element (frequency-independent resistor, capacitor or inductor) being correlated with a physical part of the transistor. Such equivalent circuits, however, suffer from one general and important disadvantage, namely that either they are valid over a limited range of frequencies or, in the attempt to cover all possible device effects, they become so complicated as to render them devoid of practical importance to the circuit designer. Obviously, each circuit element may assume widely variable values according to the D.C. operating point. Other, and perhaps more widely used methods of characterization are those which associate the transistor to a four-pole or a «black-box» with four terminals (one terminal is common to the input and the output). What goes on inside the box is immaterial and the box is characterized only by voltages and currents at the output terminals as shown in fig. 1. The general relation is :

$$f_1 (I_1, I_2, V_1, V_2) = 0$$
$$f_2 (I_1, I_2, V_1, V_2) = 0 \qquad (1)$$

and two of the variables may at will be explicitly set equal to a function of the other two. Of course, if the performance of a device having small signal swings with respect to the bias is of interest, then the relation can always be linearized by means of a Taylor expansion in a given interval whose width depends on the nature of the device and the desired accuracy of representation. As an example, Eq. (1) can be expressed as :

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$$I_1 = I_1 (V_1, V_2)$$
$$I_2 = I_2 (V_1, V_2) \qquad (2)$$

and, to a linear approximation :

$$i_1 = \left(\frac{\partial I_1}{\partial V_1} \bigg|_{v_2=0} \right) v_1 + \left(\frac{\partial I_1}{\partial V_2} \bigg|_{v_1=0} \right) v_2$$
$$i_2 = \left(\frac{\partial I_2}{\partial V_1} \bigg|_{v_2=0} \right) v_1 + \left(\frac{\partial I_2}{\partial V_2} \bigg|_{v_1=0} \right) v_2 \qquad (2a)$$

where the small letters indicate increments with respect to the starting point and the derivatives are taken at this point. The device is therefore now characterized by four quantities, the coefficients of the expansion, which must obviously be specified as a function of frequency as well as of operating point.

In the remainder of this paper, reference will be made to such linearized representations and occasionally to equivalent circuits when required.

2.0 BASIC CONSIDERATIONS

2.1 *Definition of fourpole parameter sets*

As seen in the introduction any pair of variables in Eq. (1) can be explicitly stated as a function of the other two. This gives rise to a number of possible sets of parameters, the most important being the y or «admittance» and the h or «hybrid» parameters, defined as follows :

$$\begin{cases} i_1 = y_{11} v_1 + y_{12} v_2 \\ i_2 = y_{21} v_1 + y_{22} v_2 \end{cases} \qquad (3)$$

$$\begin{cases} v_1 = h_{11} i_1 + h_{12} v_2 \\ i_2 = h_{21} i_1 + h_{22} v_2 \end{cases} \qquad (4)$$

Eqs. (3) define the «y» parameters. The meaning of the four quantities is obtained by comparison with Eq. (2a).

$$y_{11} = \frac{\partial I_1}{\partial V_1} \bigg|_{v_2 = 0}$$ = small signal input admittance with output short-circuited.

$$y_{12} = \frac{\partial I_1}{\partial V_2} \bigg|_{v_1 = 0}$$ = small signal reverse transadmittance with input short-circuited.

$$y_{21} = \frac{\partial I_2}{\partial V_1} \bigg|_{v_2 = 0}$$ = small signal forward transadmittance with output short-circuited.

$$y_{22} = \frac{\partial I_2}{\partial V_2} \bigg|_{v_1 = 0}$$ = small signal output admittance with input short-circuited.

For transistors, the four parameters are more often indicated y_i, y_r, y_f, y_o respectively, with a second subscript (e, b, c) indicating operation in the common-emitter, common-base or common-collector configuration respectively. In similar manner :

$$h_{11} = h_i = \frac{\partial V_1}{\partial I_1} \bigg|_{v_2=0}$$ = small signal input impedance with output short-circuited.

$$h_{12} = h_r = \frac{\partial V_1}{\partial V_2} \bigg|_{i_1=0}$$ = small signal reverse feed-back voltage transfer ratio with input open-circuited.

$$h_{21} = h_f = \frac{\partial I_2}{\partial I_1} \bigg|_{v_2=0}$$ = small signal forward current transfer ratio with output short circuited.

$$h_{22} = h_o = \frac{\partial I_2}{\partial V_2} \bigg|_{i_1=0}$$ = small signal output admittance with input open-circuited.

2.2 *General Matrix Relations*

The choice of which set of parameters to use is a matter of convenience and ease of measurement. As a matter of fact, the transis-

tor in the most common configuration, i.e. common emitter, has a low input impedance and a high output impedance so that it is most convenient for measurement purposes to supply the input with a constant current and the output with a constant voltage. Therefore, h parameters are generally given at low frequencies. At high frequencies, however, it becomes increasingly difficult to effectively leave an open-circuit at the input, as required for the measurement of h_r and h_o; therefore, y parameters are more easily measured and found in manufacturers' literature for medium and high frequencies. Once the four y parameters are known, the h parameters can be obtained from them and vice-versa because four parameters are necessary and sufficient to completely characterize a linear active fourpole. A list of the relations between h and y parameters is given in the Appendix.

Because of the convenience of measurement at high frequencies and also the ease of conversion from one set of parameters to the other, this paper will deal from this point on with y parameters.

Although the transistor has up to this point been considered as a fourpole, it is inherently a three-lead device. Therefore, one lead must be common to input and output, and accordingly three configurations are possible for the basic amplifier: the common emitter, the common base, the common collector.

It is easily shown that four «independent» (in some manner which will be explained later) y parameters out of the twelve possible completely specify the whole set. This is seen with reference to fig. 2. The voltages and currents at the three terminals of the linearized black box must satisfy the following equations:

$$\left|\begin{array}{l} i_e = y_{ee}\, v_e + y_{eb}\, v_b + y_{ec}\, v_c \\ i_b = y_{be}\, v_e + y_{bb}\, v_b + y_{bc}\, v_c \\ i_c = y_{ce}\, v_e + y_{cb}\, v_b + y_{cc}\, v_c \end{array}\right| \qquad (5)$$

where the meaning of each of the coefficients is obtained by inspection, putting all the voltages equal to zero except the one multiplying the coefficient of interest. For example, y_{ee} is recognized as the common-base input impedance y_{ib}, as well as the common-collector output impedance y_{oc}. Two other equalities are found in the same way ($y_{ie} = y_{ic}$; $y_{oe} = y_{ob}$), so that the matrix of coefficients of Eqs. (2.3) can be written :

$$\left|\begin{array}{ccc} y_{ib} = y_{oc} & y_{fc} & y_{rb} \\ y_{rc} & y_{ie} = y_{ic} & y_{re} \\ y_{fb} & y_{fe} & y_{ob} = y_{oe} \end{array}\right| \qquad (6)$$

By applying Kirchoff's law to the node in fig. 2, one obtains :

$$i_e + i_b + i_c = v_e\,(y_{ib} + y_{rc} + y_{fb}) + v_b\,(y_{fc} + y_{ie} + y_{fe}) + v_c(y_{rb} + y_{re} + y_{oe}) = 0$$

As the relation must hold for all values of v_e, v_b, and v_c, the three parentheses must each independently be equal to zero. In the same way it is apparent that if $v_e = v_b = v_c = v$, that is, $v_1 = v_2 = v_3 = 0$ (see fig. 2), then $i_e = i_b = i_c = 0$; therefore, from each of the (5) it follows that the sum of the coefficients of each equation must be zero. From the last two paragraphs, it may be concluded that the sum of each row and each column of matrix (6) is zero. Therefore, if four of its elements are known, three of which do not lie in the same row or in the same column, all the others can be determined by addition. For example, by repeated application of the rule to the first column and to the second and third row, the following useful relation is found :

$$y_{ib} = y_{ie} + y_{fe} + y_{re} + y_{oe} \qquad (7)$$

Of course, this procedure should be used with caution in practical cases where each of the known terms is affected by some experimental error. Care must be taken not to obtain relatively small parameters as a difference between large ones (e.g. y_{re} or y_{oe} from the other four terms of Eq. (7)) as very serious errors can result.

2.3 *Dependence of some y parameters on frequency, voltage, and current*

The aim of the following section is to give an idea of typical dependence of some y parameters on voltage, current, and frequency, with reference to basic device characteristics and to partial and approximate equivalent circuits.

2.3.1 *Basic Parameter Behaviour*

The elements to which the y parameters will be referred are y_{je}, the intrinsic admittance of the emitter-base junction; α, common-base, small signal current transfer ratio: C_c, collector depletion-layer capacitance, r_{bb}, base extrinsic or spreading resistance, and some parasitic elements (such as lead inductances and capacitances due to the package) whose importance is only relevant at high frequencies. While the extrinsic parameters are assumed to be constant, the intrinsic ones have some well-known behaviour generally dependent on transistor type which will be summarized here. The following discussion is mainly applicable to silicon double-diffused planar transistors.

2.3.1.1 *Emitter Admittance*

The emitter admittance y_{je} is made up of the emitter conductance g_e, and the emitter susceptance, which results from the emitter transition capacitance C_{TE} and the emitter diffusion capacitante C_D where $C_D+C_{TE}=C_e$.

The term g_e (equal to $1/r_e$ is proportional to emitter current and is equal to $\frac{qIe}{k\,T}$ or $1/26$ mho at 1 ma and 25°C.

The emitter transition capacitance is voltage-dependent; it can however be considered sensibily constant in the active region due to the narrow range of useful emitter bias voltages.

The emitter diffusion capacitance is the direct result of the diffusion mechanism of charge transport. In alloy (no drift-field) transistors a gradient of electric charge is necessary to transport minority carriers from emitter to collector; this charge gradient must be provided by additional minority carriers which have to be injected into the base before transistor action can take place. As the injection of carriers from the emitter requires a change of voltage across the junction, a capacitance (diffusion capacitance) can be defined as the ratio of change of injected charge into the base to the change of voltage (fig. 3). In drift (diffused-base) transistors, however, the main mechanism of charge transport is by drift in the electric field due to the doping gradient so that the diffusion capacitance is smaller. In any case, the diffusion capacitance is current dependent in the same way as the emitter conductance. The ratio of conductance to capacitance (i.e. $\frac{g_e}{C_e}$ or $\frac{1}{r_e\,C_e}$ which has the dimension of frequency) is therefore independent of emitter current to a first approximation and is a measure of the high-frequency capabilities of the transistor; it is, in fact, proportional to the $f_{\alpha b}$ of the transistor.

2.3.1.2 *Current Gain*

The common base current gain, α, has a behaviour at low frequencies resulting from effects such as emitter efficiency, recombination within the base and the emitter junction and collector efficiency. These factors normally

combine to give an α which remains slightly less than 1 in a wide range of operating conditions; the fraction $(1 - \alpha)$ of emitter current flowing through the base lead, however, is much more sensitive to operating conditions than α itself; although $1 - \alpha$ has a broad minimum (particularly broad for the protected-junctions planar devices), its magnitude increases at low currents due to trap-assisted recombination in the base and in the emitter - base junction, and at high currents due to a decrease in emitter efficiency.

The high frequency behaviour of α is primarily affected by the presence of the emitter capacitances. Qualitatively speaking, at high frequencies, an increasing fraction of the charge injected during one cycle of the input signal flows through the base lead; this charge is continuously modifying the charge distribution within the base necessary for transistor action at expense of the charge flowing to the collector as a result of transistor action itself. This is the same as saying that at high frequencies α is reduced in magnitude and delayed in phase and may be accounted for approximately by the equation :

$$\alpha = \alpha_o \frac{e^{-j\nu f/f_{\alpha b}}}{1 + j\frac{f}{f_{\alpha b}}} \qquad (8)$$

where α_o is the low-frequency α, f the frequency, $f_{\alpha b}$ the α cut-off frequency and ν a phase delay factor taking into account the presence of the drift field into the base resulting from the doping gradient in diffused-base transistors (to a first approximation $\nu = 0$ in homogeneous base transistors and typically $\nu = 0,5$ to 1 in planar transistors).

It should be noted that since $1 - \alpha$ is much more sensitive to changes in frequency than α itself, it happens that much before α has appreciably changed its magnitude, it has accumulated a phase angle sufficient to make $1 - \alpha$ much larger than it is at zero frequency (fig. 4). As a function of frequency, one has

$$1 - \alpha \simeq \frac{1 - \alpha_o + j\frac{f}{f_{\alpha b}}(1 + \nu)}{1 + j\frac{f}{f_{\alpha b}}} \qquad (8a)$$

The first singularity is due to the numerator which occurs at

$$f = f_{\alpha b}\frac{(1 - \alpha_o)}{(1 + \nu)}$$

The frequency at which $1 - \alpha$ is 3 db above its low frequency value is therefore $f_{\alpha b}(1-\alpha_o)/(1 + \nu)$. According to (8a) $1-\alpha$ increases 20 db per decade at frequencies above the first singularity (this rule is remarkably well followed in practice); if f_t is the frequency at which the base current has increased in magnitude so as to become equal to the collector current (common emitter current gain equal to 1), then one has :

$$f_{\alpha b} \simeq (1 + \nu) f_t$$

2.3.1.3 *Collector Capacitance*

The collector depletion - layer capacitance C_c, or C_{ob} (approx.), is primarily a function of collector-base voltage; higher voltages widen the depletion - layer and lower the capacitance according to a $V_{cb}^{-1/3}$ law, in the case of diffused junctions.

2.3.2 *y Parameter Behaviour*

Starting from the above mentioned transistor parameters, the behaviour of some four-pole y parameters will now be discussed. As a general rule, such behaviour will be more complex for the y parameters than for the device parameters

because each y parameter as observed from the outside is affected simultaneously by several device mechanisms.

2.3.2.1 *Common base input admittance* y_{ib}

Neglecting feed-back currents from collector to emitter, which are generally small with respect to input current, the equivalent circuit for the input part of the transistor is shown in fig. 5a.

At low frequency, the main part of the input current of a common-base amplifier flows from emitter into the collector, so the influence of $r_{bb'}$ on y_{ib} is reduced by a factor $1/(1-\alpha)$ since the current through $r_{bb'}$ is reduced by that factor; i.e. :

$$y_{ib} = \frac{1}{\frac{1}{y_{je}} + r_{bb'}(1 - \alpha)} \qquad (9)$$

At low frequencies, therefore, except for very high collector currents and very low current gains, the input admittance is essentially g_e. At higher frequencies, however, $1 - \alpha$ first rotates in phase and then increases in magnitude. When the dependence of α on frequency is introduced in (9), it is found that with increasing frequency y_{ib} takes an inductive component, while the real part decreases, tending towards $1/r_{bb'}$ for very high frequencies. This is shown in fig. 6 where a polar plot of y_{ib} is drawn for the simple case of α obeying Eq. 8 (solid line: $\nu = 0$).

The reason for such behaviour can be simply explained intuitively. If the input is thought to be a voltage step instead of a continuous h-f signal, one sees that immediately after application of the step, more current flows through the base and through $r_{bb'}$ than in the steady-state, until the required charge distribution is established within the transistor. For the very initial part of the signal, containing very high frequency components, the impedance seen is $r_{bb'}$ since transistor action has not yet begun. For the following parts of the applied step, current is shared between base and collector, the part taken by the collector increasing with time and the sum increasing too, as the influence of $r_{bb'}$ on y_{ib} decreases. But this is exactly the step response of the L-R circuit of fig. 5b which as far as the input is concerned, is equivalent to fig. 5a aside from the substitution of an inductance for the collector branch and its effects. It should now be added that the excess-phase approximation of α brings about a modification of the polar plot in fig. 6 shown by the dotted line. Another change is introduced by parasitic elements, particularly the inductance of the emitter lead; it can be shown that the effect of this inductance is mainly a reduction of the real part of y_{ib}, the reduction increasing with frequency (see later).

2.3.2.2. *Common Emitter Input Admittance* y_{ie}

Reference is made again to fig. 4 although interest if now focused on base current (fraction $1 - \alpha(f)$) of emitter current.

Here again, it could be said that at low frequency the base input impedance is the sum of $r_{bb'}$ and of the emitter junction resistance multiplied by $1/(1 - \alpha)$, because any current through the base is associated in the emitter junction, to a current $\frac{1}{1 - \alpha}$ times larger flowing to the collector. Thus, common emitter input conductance is much smaller than with common-base .

As frequency increases, base current increments are in a large range in quadrature with emitter current because the magnitudes of emitter and collector currents remain about the same while their phase difference increases with

frequency (this is especially true for the diffused-base transistor); the base current can therefore be thought as due to a capacitor $C_{b'e}$ connected in parallel with the input conductance. This R-C circuit determines the cut-off frequency in the common-emitter connection. From eq (8a), this cut-off frequency is equal to $f_{\alpha b}\frac{(1-\alpha_o)}{1+\nu}$ and is the frequency at which the common-emitter current gain is 3 db below its 1 - f value.

The RC time constant is therefore $(1 + \nu)/(1 - \alpha_o)$ times larger than with common-base. The polar plot pertaining to y_{ie} (completed with the addition of $r_{bb'}$) is shown in fig. 7.

All the elements in the equivalent circuit of fig. 5a can in principle be obtained from such a polar plot.

y_{ie} is affected in the same manner as y_{ib} by the lead impedances, particularly the emitter lead which carries the sum of the base and collector currents. This effect is seldom important below 100 Mc.

2.3.2.3 *Common emitter forward transfer admittance* y_{fe}

By definition, $y_{fe} = \left.\frac{\partial I_c}{\partial V_{be}}\right|_{v_{ce}=0}$

its order of magnitude is thus the same as y_{ib} (see also Eq. 7). The transfer admittance for an intrinsic transistor ($r_{bb'} = 0$), y_{fe}^*, has a frequency dependence which can be estimated by considering that, v_{be} being the input variable, the effects of the input capacitances are excluded from y_{fe}^* since a voltage source exists and only an excess phase factor or delay would affect y_{fe}^* at high frequency. The presence of $r_{bb'}$ however, upsets these considerations because of the voltage division between $r_{bb'}$ and the intrinsic input impedance (fig. 8). This appreciably departs from one when the reactance of $C_{b'e}$ approaches the parallel impedance of $r_{bb'}$ and $r_{b'e}$ (the latter is frequency dependent) This generally happens at some frequency larger than $f_{\alpha b}(1 - \alpha)$.

2.3.2.4 *Common Emitter Reverse Transfer Admittance* y_{re}

This is the ratio of A.C. current through the base lead (and base spreading resistance) to the A.C. collector excitation voltage. At very low frequency (under, say, 1 kc) y_{re} is mainly conductance and is due to the effect of collector space-charge layer widening: the varying collector voltage modulates the width of the collector barrier and therefore of the base, so that the base transport factor and the emitter efficiency and therefore α and $(1 - \alpha)$ change. Further, variation of the base width affects the concentration gradient of minority carriers at the emitter side of the base, which in turn affects the emitter-diode V-I relationship in a manner which can be represented by a resistor r_c from base to collector (fig. 9). This is in parallel with another resistor due to surface leakage through the c-b junction and to external leakage in the package. These effects were important in early types of transistors, but are almost always negligible in the more recent types having oxide-protected junctions.

In these types only the «intrinsic» effect is usually present and it, as well, is quite small on its own; while conductances are often below 1 μmho, they are frequently swamped by the admittance due to the collector capacitance even at moderately low frequencies. Thus, in a large range of frequencies above a few kilocycles, y_{re} is predominantly imaginary. At higher frequencies, $r_{bb'}$ is no longer negligible with respect to $\frac{1}{\omega C_{b'e}}$ and $\frac{1}{\omega C_c}$, so that a real compo-

nent is developed as a result of current partition between $r_{bb'}$ and $C_{b'e}$ and of the finite reactance of $C_{b'e}$.

2.3.2.5 *Common Emitter Output Admittance* y_{oe}

The analysis of y_{oe} shows the convenience of splitting the transistor into two parts, one made up of r_c, C_c, $r_{bb'}$, $C_{b'e}$ and the input resistive component and constituing an R-C network, the output of which is connected to the second part, the voltage-controlled generator (see fig. 10). It is seen that due to the presence of $r_{bb'}$ the input short-circuit is quite ineffective, so that some part of the collector voltage is present between terminals b' and e; this gives rise to a current in the generator which adds to that flowing in the network and thus modifies the output admittance due to the passive terms alone. For reasons shown previously for y_{re}, it happens in most cases that the time constant associated with the collector is larger than that associated with the base-emitter circuit. At low frequency the base-emitter voltage, therefore, leads the collector-emitter voltage by 90° and the same is true for the current $y_{fe}^* v_{be}$ (the frequency is assumed not to be so high that the intrinsic transconductance y_{fe}^* has appreciably changed from the D.C. value). The current in the generator has thus the same phase as the current which would flow in a capacitor, or, in other words, the effect of the feed-back due to $r_{bb'}$ is the same as increasing the output capacitance. It can be shown that the output capacitance has a value $C_c (1 + y_{fe}^* r_{bb'})$; if $r_{bb'}$ is smaller than the other impedances in the input branch, the effect of r_c can be neglected and y_{fe} can be assumed real. At higher frequencies, the first and third assumptions tend to be invalid, so that a real component of y_{oe} is developed and the «apparent» capacitance drops until, at a high enough frequency, only the contribution of the elements in the upper part of fig. 10 is appreciable.

3.0 MEASUREMENT TECHNIQUES

The experimental part of this investigation called for the small-signal characterization of transistors in a frequency range from low radio frequency (where most of the parameters of high-quality planar transistors are still essentially real) to as high a frequency as practicable. The boundary conditions were also fixed by the instruments available; these were the Wayne Kerr bridges B-601 (50 Kc to 5 Mc) and the B-801 (1 to 100 Mc).

3.1 *Bridge Characteristics*

These instruments are transformer ratio-arm bridges, permitting 2- and 3- terminal measurements; the basic concept of such bridges is shown in fig. 11. An input transformer with a center tap, fed by a signal generator, forces currents i_1 and $-i_2$ respectively into a standard admittance (made up of a real and an imaginary part) and into the A-lead of the unknown. Some current i_o then flows in lead B ($i_o = i_2$ for two terminal measurements as terminal N (neutral) is left open in this case). When the standard admittance is adjusted in such a way as to have $i_o = i_1$ then no current flows in the primary of the output transformer and therefore no signal is detected and the bridge is balanced. The standard admittance is then equal to the admittance from A to B (when N is open) (for two terminal measurements) or to the transadmittance from A to B, when measuring three terminal networks. Multiple taps can be provided on the output transformer (the balance condition is such as to have an equal number of ampereturns in its primary winding due to the standard and the unknown admittances), so that the range of measurement can be very wide (e.g. 0.1 μmho to 0.1 mho for the real part in the B-601 bridge).

Some peculiarities in the method of measurement are at once apparent. These are :

1) Two terminal measurements can be performed in presence of large admittances from A to N and B to N. The former do not influence the balance condition as long as the additional current i_{AN} in the input transformer does not alter the voltage ratio as a result of leakage inductance; the latter does not alter the balance ($V_{B-N} = O$ at balance), but reduces the sensitivity of balance only.

2) As a result of point 1, paths can be provided for bias currents without the measurements being affected.

3) The most directly measurable transfer parameters are y parameters.

4) When measuring transfer admittances, the bridges are only capable of being balanced when the real part of the unknown transadmittance is of such a sign as to make a current flow out of terminal B for a positive voltage at A, hence, only negative transconductances (in the sense of sect. 1) can be measured directly.

5) Negative inductive susceptances or transsusceptances can be balanced by considering them equivalent to negative capacitors.

3.2 *Bridge Adaptors and Simplified Circuits*

Due to point 1 and in the interests of standardization, the adaptors were designed for measurement in the common-emitter connection; y_{fe}, however, has a positive real part in most of the useful range of frequencies so that it is not directly measurable due to point 4 above. This difficulty is removed by the insertion of a phase reversing transformer for the B-601 bridge adaptors; for the B-801, the measurement of y_{ib} is made instead of y_{fe}, and y_{fe} is calculated from eq. (7). Other useful relations between y_e, y_b and h parameters are listed in the Appendix. Equivalent circuits for measurements of the four y_e parameters on the B-601 bridge are shown in fig. 12. Equivalent circuits for the B-801 bridge adaptors are similar, except for y_{fe}, which is replaced by y_{ib} (see fig. 13).

In all the circuits the leads to the bridge, some of which include by-pass condensers, require a low series impedance over the whole range of measurement, while the arms in parallel with the input and the output should satisfy the conditions stated in point 1 above, and still be relatively low-resistance paths for D.C. The major sources of error are analyzed below.

3.3 *The analysis of Possible Errors*

3.3.1 *Errors Due to Series Lead Impedance*

The major source of errors is due to connections to the bridge, i.e. conductors and by-pass condensers; the latter, depending on frequency range and on type, may appear other than capacitive. Pairs of back-to-back-connected 22 μF tantalum electrolytic capacitors are used on the B-601 jigs (the back-to-back connection allows both PNP and NPN transistors to be measured). This would mean about 0.3 ohms series reactance at 50 Kc which would be adequate in practical cases. These condensers, however, like all electrolytic condensers, exhibit some series resistance, which begins to be appreciable from about 10 Kc on (see fig. 14); hence, in places where series impedance is particularly important, electrolytic capacitors are paralleled by 2 μF synthetic film condensers.

The higher working frequencies of the B-801 make lead inductance important; this can be appreciated by considering that a length of 2 cm. of straight 0,6 mm wire offers about 8 ohms reactance at 100 mc. Therefore, it is important

to minimize lead length by a careful layout and to substitute short large area straps for wire connections; critical by-pass capacitors were built by paralleling three 0.01 μF disc ceramic condensers whose leads were totally removed and soldering their electrodes directly on a thin brass sheet (see fig. 39). These prove to be exceedingly effective.

In general, it should be pointed out that series lead inductance is much more detrimental to performance than capacitance both to the neutral and between A and B terminals; the former is of no consequence provided point 1) of sect. 3.1 is considered; the latter can be balanced out during the initial bridge adjustment and has in many cases no other effect. It pays therefore to try to reduce inductance even at the expense of increased capacitance, e.g. by having wide connections in the immediate vicinity of ground planes; this is obtained without too much difficulty by using printed circuit techniques.

Some series inductance is unavoidable, however, and this is most objectionable for high conductance measurements, in particular for y_{ib}. A quantitative evaluation of this effect was undertaken with reference to the input equivalent circuit of fig. 15 where L_1 is the lead inductance and L_2 and G represent the inductive and conductive intrinsic values of the unknown (L_2 and G are not frequency independent, of course). An analysis of the circuit gives the following results :

$$\frac{G}{G_O} = \frac{(1-\beta)^2}{1+(\omega T_2 \beta)^2}$$

$$\frac{B}{B_O} = (1-\beta)\frac{1+\beta(\omega T_2)^2}{1+(\beta\omega T_2)^2}$$

where G_O and B_O are the intrinsic values of the unknown, G and B the actual measured values, $\beta = \frac{L_1}{L_1 + L_2}$, $T_2 = L_2 G$, ω the angular frequency of measurement.

Such analysis has been compared with experiment by performing two measurements on the same specimen in the same conditions, but with the transistor plugged into the socket with different lead lengths. Emitter lead inductance was assumed to have predominant effect and its value was estimated by means of tables. Results are summarized below.

2N 916, 10 V 5 mA

f mc	L_1 mH (est.)	L_2 mH	G_o	β	T_2 nsecs	$\frac{G}{G_o}$ calculated	$\frac{G}{G_o}$ measured
40	5	97.5	43.3	0.05	4.20	0.902	0.91
50	5	91	36.8	0.053	3.34	0.892	0.91
75	5	74	28.6	0.063	2.11	0.871	0.88
100	5	62	24.4	0.075	1.51	0.845	0.87

A considerable reduction in the effects of series lead impedances can also be gained by careful choice of the jig circuit. Referring to fig. 12a it should be pointed out that while C_1 could be returned to point B instead of to the emitter, this connection is to be avoided because it allows the full A.C. emitter current to flow through C_2 instead of the base current. Thus, the effect on measured values is much larger, particularly when ωC_2 is not much larger than the emitter admittance.

Similar considerations can be made for the other jigs.

3.3.2 *Errors Due to the Arms in Parallel with the Bridge Transformers*

Another general source of error is represented by A.C. currents flowing through these arms; they may produce feedback at the measuring

frequency as well as at other frequencies. This problem was especially troublesome with the B-601 bridge, when measurements were required to be possible with collector currents up to 100 mA. As the minimum impedance to be connected from terminals A and B to N was experimentally found to be around 1000 ohms, a value of 5000 ohms was chosen to provide a reasonable safety factor; therefore, large power dissipation would result if resistors were used, and this suggested the use of small 5 mH ferrite inductors. When placed in parallel with the bridge transformer windings, these proved to have no appreciable influence on the measurement of passive components even at the lowest frequency of interest, i.e. 50 kc. These inductors, however, introduced large feedback at low frequencies when used in the jigs with the transistor under test, and indeed the jigs were found to oscillate at about 1 kc. unless the inductors were heavily damped by series resistors.

By examination of fig. 12a, indeed the collector inductor and the two by-pass condensers can be recognized as a Colpitts oscillator.

Feedback at frequencies within the range of measurement is usually more troublesome, even if not large enough to sustain oscillation. This is particularly true if one of the admittance components is much smaller than the other, such as b_{ie} (imaginary part of y_{ie}) at low frequency, so that even a relatively small error in y_{ie} can seriously affect b_{ie}.

3.4 *Experimental Set-up and A.C. Level Adjustment*

Some considerations are now required regarding the whole measurement set-up. This is made up of a signal generator, bridge with adaptor, and null detector as in fig. 16.

Obviously, direct transmission from generator to detector (not through the bridge circuits) is to be avoided because this added path provides an unwanted signal which combines (vectorially) with the true signal; a false balance is therefore observed.

Extreme care should be taken in shielding the generator from the detector as well as the connections especially at the higher frequencies. In any case, the degree of shielding can be checked by disconnecting the generator or detector and observing the residual signal.

The signal level at the measurement terminals should not be so large as to invalidate the small-signal assumption. This is dependent on the parameter to be measured and on D.C. conditions and frequency; this may be understood by reference to fig. 5. Frequency and collector current affect the partition ratio and the emitter impedance, therefore larger A.C. signals are allowed at high frequency and/or at high current. As a general rule, measurements on non-linear parameters require the observation of the null position as a function of A.C. signal level; a suitable level is found when a further decrease does not appreciably change the position of the null. The word «appreciably» means in practice that a compromise has sometimes to be found with the requirement of adequate sensitivity. As a rule of thumb, suitable levels at the transistor terminals are :

- 5 to 10 mV rms for measurements of y_{ib}, y_{ie}, y_{fe}
- about 500 mV rms for measurements of y_{re}, y_{oe}

An example of determination of suitable signal level is shown in fig. 17; it is seen that both g_{ie} and b_{ie} can be considered constant for signal levels below 8 - 10 mV.

Of course, adequate sensitivity from the null detector is in the microvolt range.

4.0 MEASUREMENTS

4.1 *Low frequency measurements*

(50 Kc/s, 5 Mc/s)

As previously mentioned, four adaptors were built for measurements with the B-601 bridge· with them, the four y_e parameters are directly measurable.

4.1.1. *Circuit diagrams*

These are shown in fig. 18 to 21. The main features were already discussed in Sect. 3. The diodes connected in parallel with the RF chokes serve as an open circuit for low levels (A.C. signal), but clip off the transients which are developed when a transistor is removed from the socket with applied bias. These transients were often found to be a cause of transistor failure.

The transformer used in the y_{fe} adaptor must fulfill the requirements of low leakage inductance together with high shunt inductance. The former is most troublesome at high frequency as it is effectively in series with the base lead and thus contributes to the «extrinsic» part of the input admittance and to the partition ratio mentioned in Sect. 2.3.2.3. The latter must only be so high as not to invalidate point 1 in Sect. 3.1. One of the adaptors (y_{fe}) is shown in fig. 22.

4.1.2. *Measurements*

The four y_e parameters and h_{fe} were measured as a function of frequency for two transistor types (2N 1613 and 2N 916) each for a typical value of applied bias. The former is a general purpose npn silicon planar transistor useful to about 30 mc. The latter is also an npn silicon planar transistor useful to about 100-200 mc. Data were also obtained for a 2N2303 which is complementary to the 2N1613. Some data are summarized in fig. 23 to 26. The accuracy of results has been checked when possible with other instruments; in particular with an h parameter tester at low frequency and with the B-801 adaptors at high frequency.

4.2 *High Frequency Measurements*

Measurements of y_{ib}, y_{ie}, y_{re}, y_{oe} were performed from 1 to 100 mc on adaptors designed for the B-801 bridge. Diagrams of the four adaptors are drawn in fig. 27 to 30. Adaptors are built on small double sided printed circuit boards. These ensure broad and short connection to each electrode. The socket is wedge mounted so that clipping of the transistor leads is not necessary for testing, while the plane of measurement is only a few mm. away from the case.

The inductance of the by-pass condensers is kept low with the «sandwich» construction mentioned in Sect. 3.3.1 (see fig. 31).

For these adaptors, too, measurements were taken as a function of frequency and results are shown in fig. 32 to 35.

Low frequency checks were made with the B-601 bridge and high frequency ones with the General Radio 1607-A transfer function and immittance bridge.

5.0 CONCLUSIONS

The techniques required for the measurement of transistor y parameters throughout a very wide range of frequencies have been described and the important practical points to follow have been stressed. It is felt that this information is sufficient for those who wish to have a practical feel for these measurements.

6.0 APPENDIX

h- and y- matrix parameters are related in the following way :

		y_i y_f	y_r y_o	h_i h_f	h_r h_o
y_i	y_r			$\frac{1}{h_i}$	$-\frac{h_r}{h_i}$
y_f	y_o			$\frac{h_f}{h_i}$	$\frac{\Delta h}{h_i}$
h_i	h_r	$\frac{1}{y_i}$	$-\frac{y_r}{y_i}$		
h_f	h_o	$\frac{y_f}{y_i}$	$\frac{\Delta y}{y_i}$		

where $\Delta h = h_i h_o - h_r h_f$, $\Delta y = y_i y_o - y_r y_f$

These approximate relations among common-emitter and common-base h parameters are also often useful :

$$h_{ib} = \frac{h_{ie}}{1 + h_{fe}}$$

$$h_{rb} = \frac{h_{ie} h_{oe}}{1 + h_{fe}} - h_{re}$$

$$h_{fb} = -\frac{h_{fe}}{1 + h_{fe}}$$

$$h_{ob} = \frac{h_{oe}}{1 + h_{fe}}$$

7.0 LIST OF THE FIGURES

1) Basic fourpole
2) Isolated three-terminal device
3) Minority carrier distribution a)alloy-junction transistor (no built-in field);b) diffused-base transistor (with built-in field)
4) Polar plot of α and 1-α vs. frequency
5) Input equivalent circuit
6) Polar plot of y_{ib}
7) Polar plot of y_{ie}
8) Partial equivalent circuit for y_{fe}
9) Partial equivalent circuit for y_{re}
10) Partial equivalent circuit for y_{oe}
11) Ratio arm bridge
12) Jigs schematic diagram for y_e measurement
13) y_{ib} measurement
14) Series impedance of chosen by-pass condensers
15) Equivalent input circuit showing parasitic emitter inductance
16) Bridge set-up
17) Example of y_{ie} measurement as a function of the A.C. rms signal
18) Circuit for y_{ie} measurements
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21) Circuit for y_{oe} measurements
22) Close-up view of y_{fe} jig for low frequency
23-24-25-26) y Parameter measurements for a 2N 2303 transistor
27) Circuit for y_{ib} measurements
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31) Close up view of y_{re} jig for high frequency
32-33-34-35) Measurements of y parameters at high frequency

LIST OF SYMBOLS

C_c = collector depletion-layer capacitance
C_D = emitter diffusion capacitance
C_e = $C_D + C_{TE}$ = total emitter capacitance
C_{TE} = emitter transition capacitance
f = frequency
f_t = frequency at which the common emitter forward current gain $h_{fe} = 1$
$f_{\alpha b}$ = common-base cut-off frequency

G = y_{ib} intrinsic conductance

g_e = $\frac{1}{re}$ = emitter intrinsic conductance

h_{11} = h_i = small signal input impedance (output short-circuited)

h_{12} = h_r = small signal reverse voltage transfer ratio (input open-circuited)

h_{21} = h_f = small signal forward current transfer ratio (output short-circuited)

h_{22} = h_o = small signal output admittance (input open-circuited) (The second subscript: e,b,c, indicates operation in the common-emitter, base or collector respectively).

I = steady state current

i = small signal current (The subscript 1, 2 is for input or output; the subscript e,b,c, is for emitter, base, or collector respectively)

k = Boltzmann's constant

$\frac{kT}{q}$ = thermal voltage (about 26 mV at room temperature)

L_1 = series parasitic inductance in y_{ib} measurement

L_2 = y_{ib} intrinsic inductance

q = negative electronic charge

$r_{bb'}$ = base extrinsic resistance

r_c = collector intrinsic resistance

r_e = emitter intrinsic resistance

T = temperature in °K

T_2 = $L_2 G$ = intrinsic time constant of y_{ib} admittance

V = steady state voltage

v = small signal voltage (The subscript 1,2 is for input or output; the subscript e, b, c is for emitter, base or collector respectively)

y_{fe}^* = common emitter transfer admittance for an intrinsic transistor ($r_{bb'}$=o)

y_{je} = intrinsic admittance of the emitter base junction

y_{11} = y_i = small signal input admittance (output short-circuited)

y_{12} = y_r = small signal reverse transadmittance (input short-circuited)

y_{21} = y_f = small signal forward transadmittance (output short-circuited)

y_{22} = y_o = small signal output admittance (input short-circuited) (The second subscript e,b,c, indicates operation in the common-emitter, base or collector respectively)

α = common base current gain (forward current transfer ratio with output short-circuited)

α_o = low frequency α

β = $\frac{L_1}{L_1+L_2}$ = dimensionless constant for series parasitic inductance error determination in y_{ib} measurement

ν = proportionality constant for the «excess phase» in α

ω = angular frequency.

8.0 REFERENCES

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4) Pettit, J.M. and McWhorter, M.M. Electronic Amplifier Circuits, McGraw - Hill

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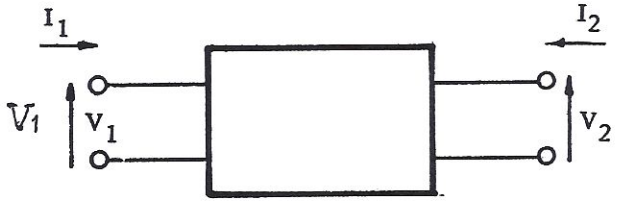


Fig. 1 - Basic fourpole

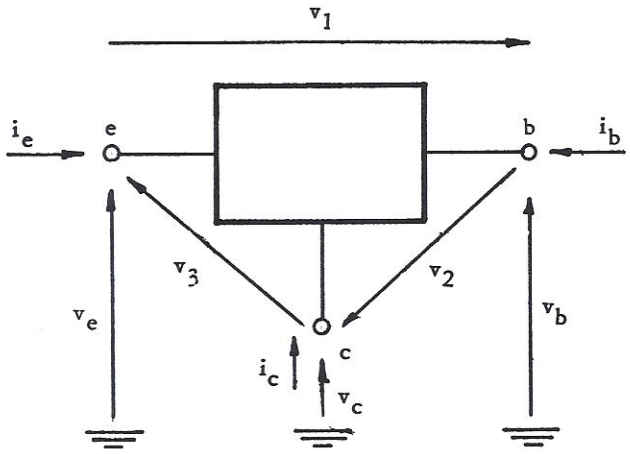


Fig. 2 - Isolated three terminal device

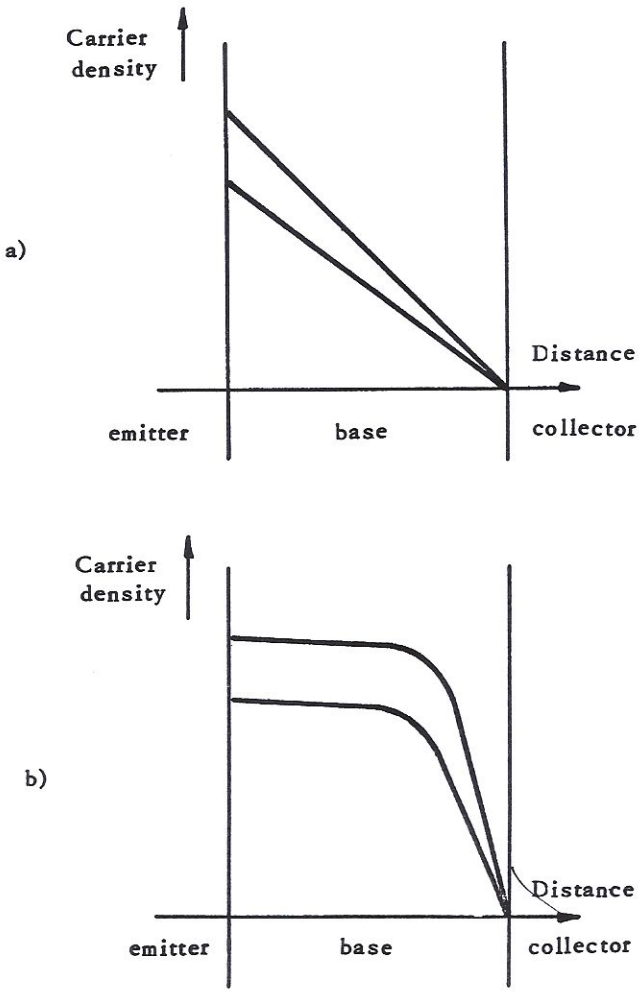


Fig. 3 - Minority carrier distribution (NPN)
a) Alloy-junction transistor (no built - in field)
b) Diffused-base transistor (with built - in field)

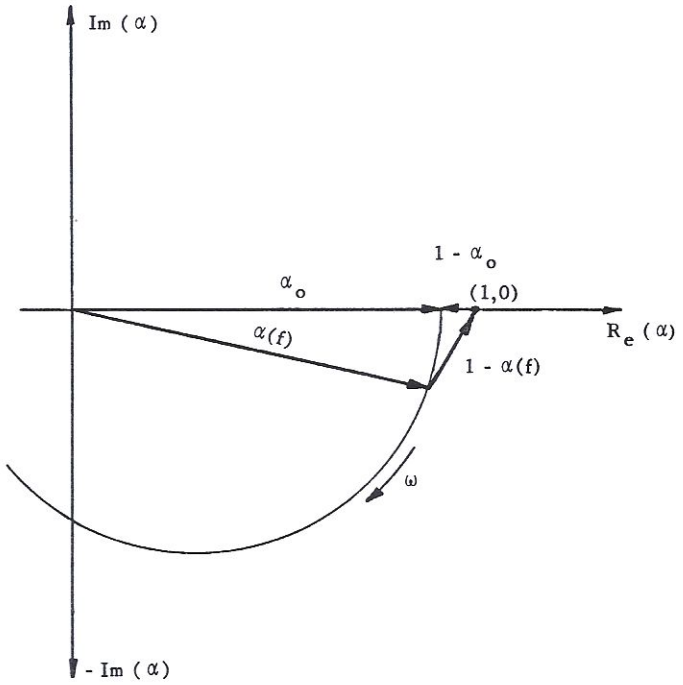
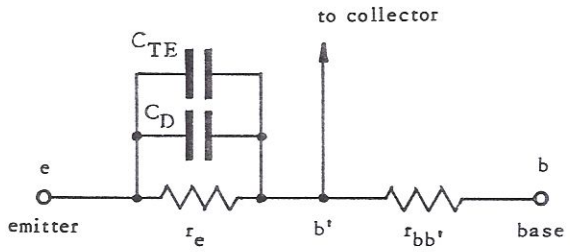


Fig. 4 - Polar plot of α and (1 - α) vs. frequency

a)



b)

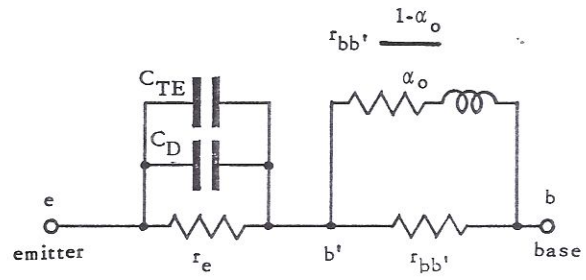


Fig. 5 - Input equivalent circuit,

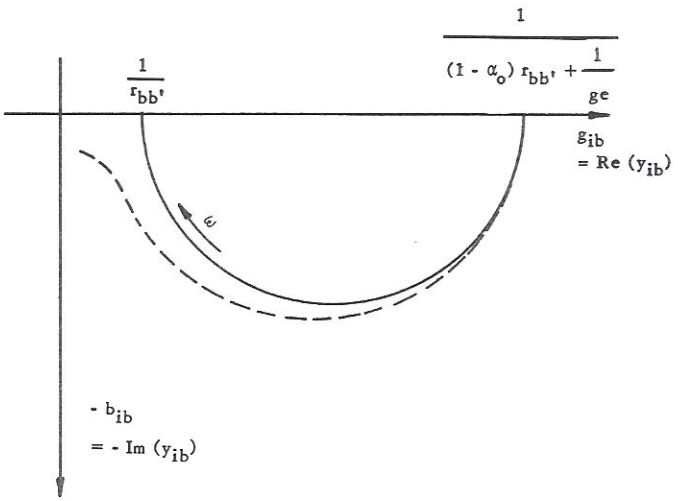


Fig. 6 - Polar plot of y_{ib}

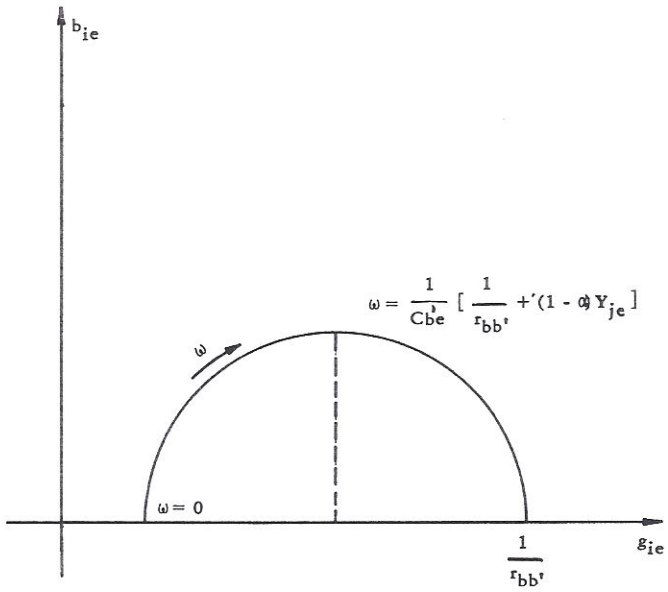


Fig. 7 - Polar plot of y_{ie}

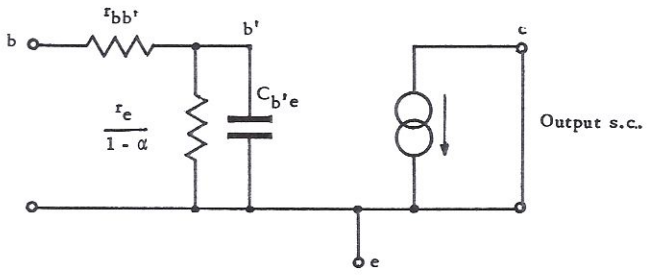


Fig. 8 - Partial equivalent circuit for y_{fe}

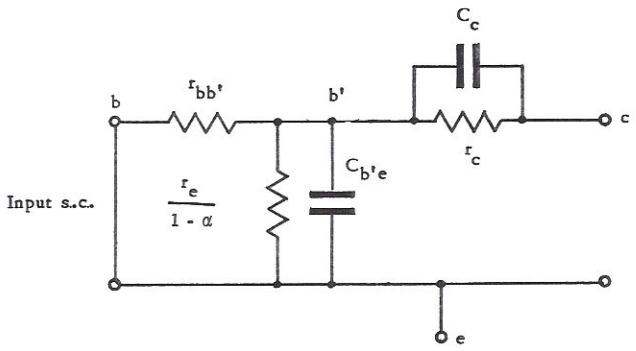


Fig. 9 - Partial equivalent circuit for y_{re}

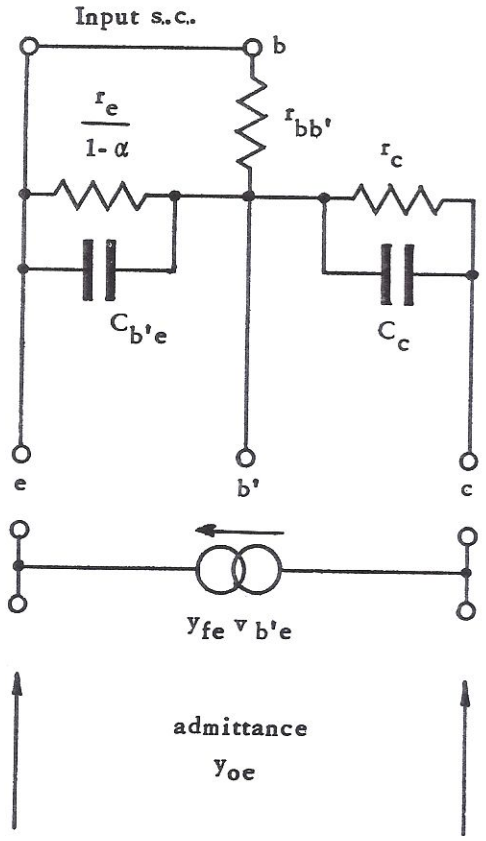


Fig. 10 - Partial equivalent circuit for y_{oe}

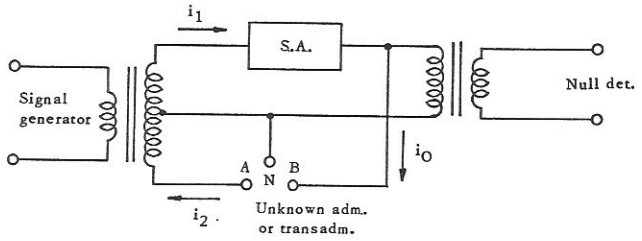
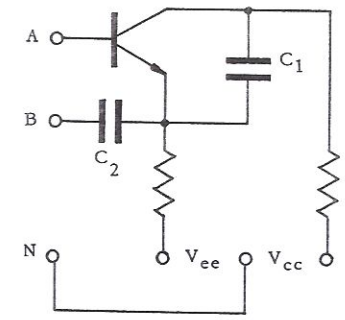
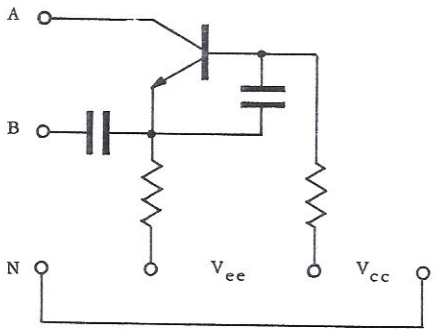


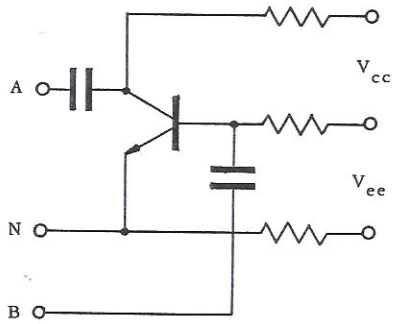
Fig. 11 - Ratio arm bridge



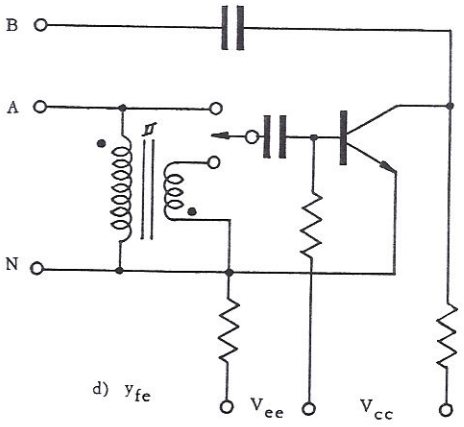
a) y_{ie}



b) y_{oe}



c) y_{re}



d) y_{fe}

Fig. 12 - Jig schematic diagram for y_e measurement

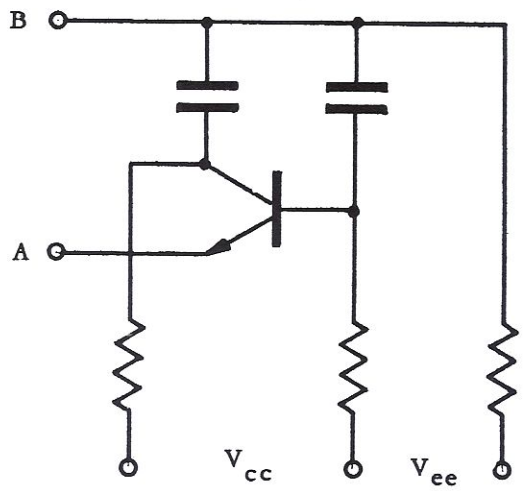


Fig. 13 - y_{ib} measurement

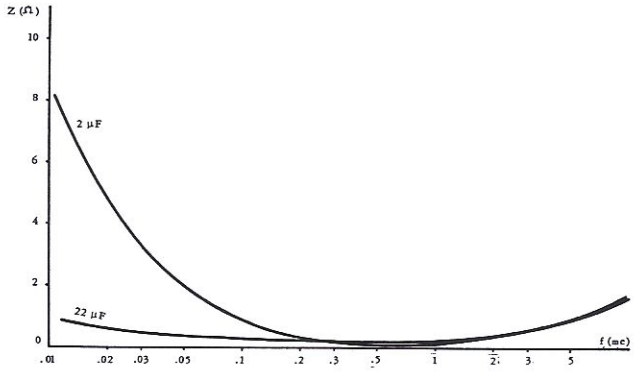


Fig. 14 - Series impedance of chosen by-pass condensers

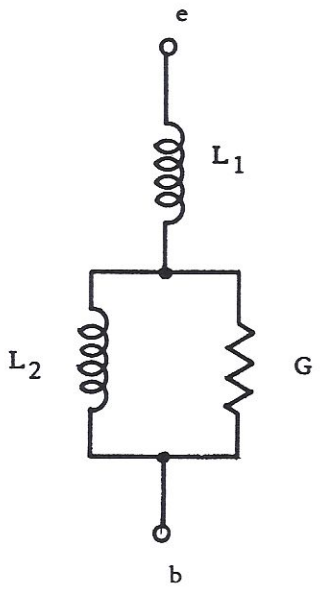


Fig. 15 - Equivalent input circuit showing parasitic emitter inductance (L_1)

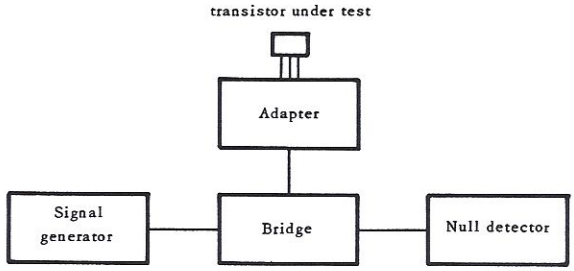
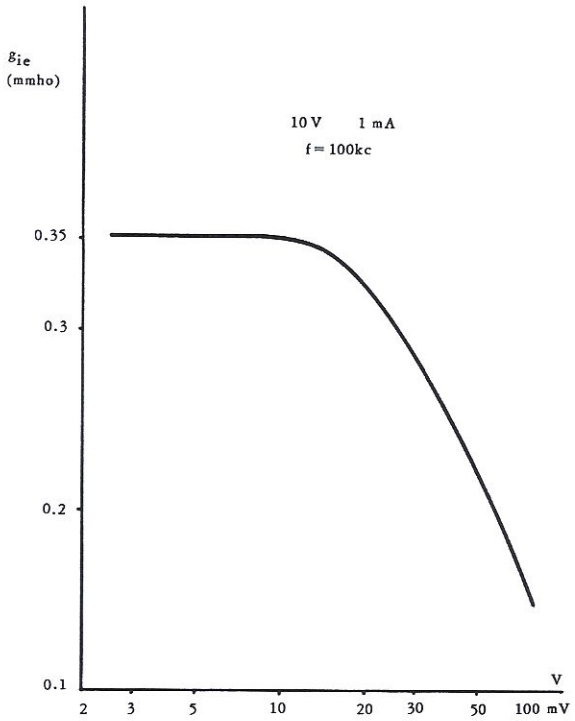


Fig. 16 - Bridge set up



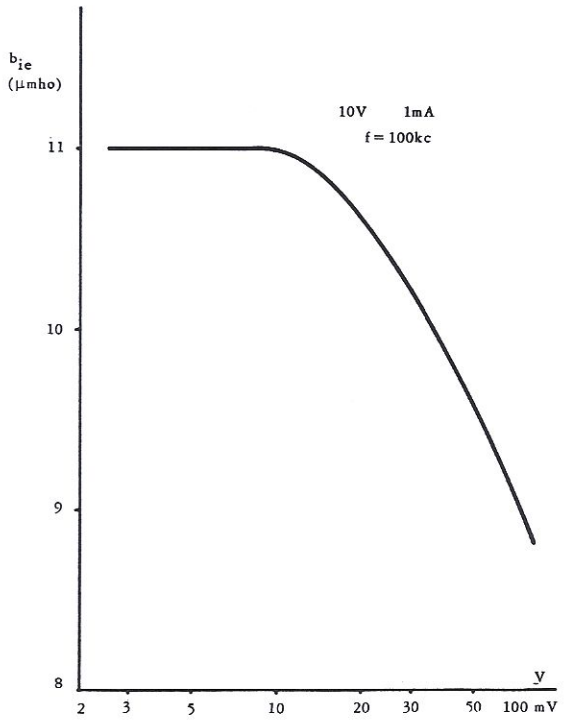


Fig. 17 - Example od y_{ie} measurement as a function of the a.c. RMS signal

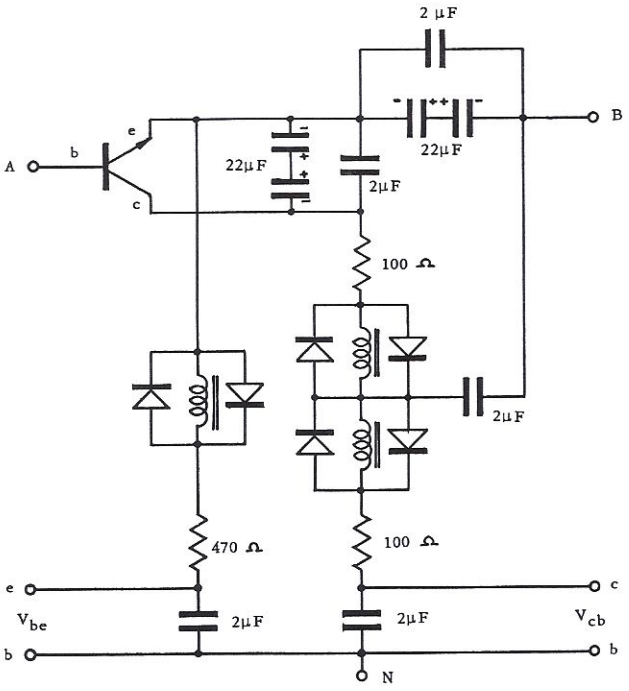


Fig. 18 - y_{ie} jig

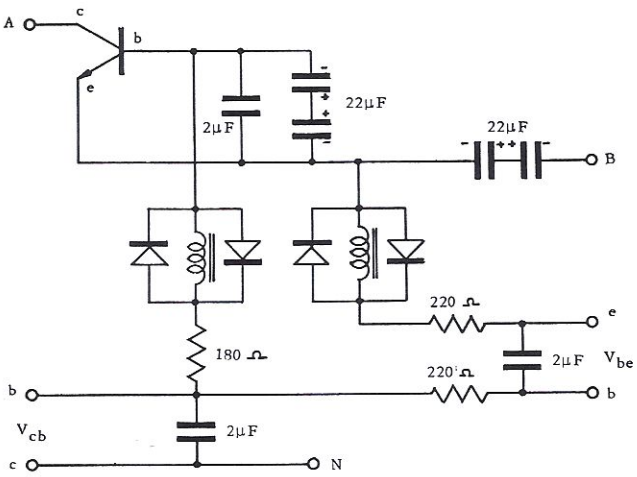


Fig. 21 - y_{oe} jig

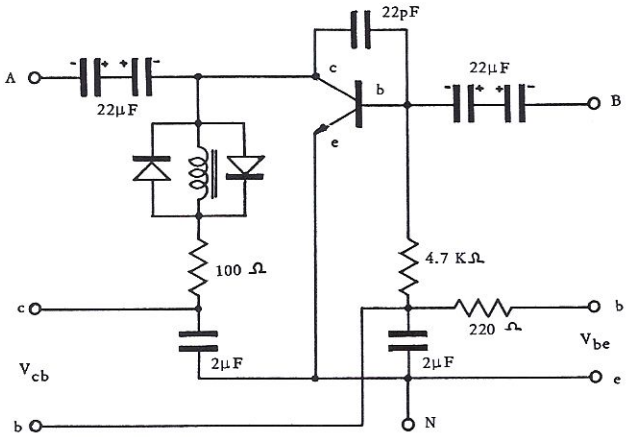


Fig. 19 - y_{re} jig

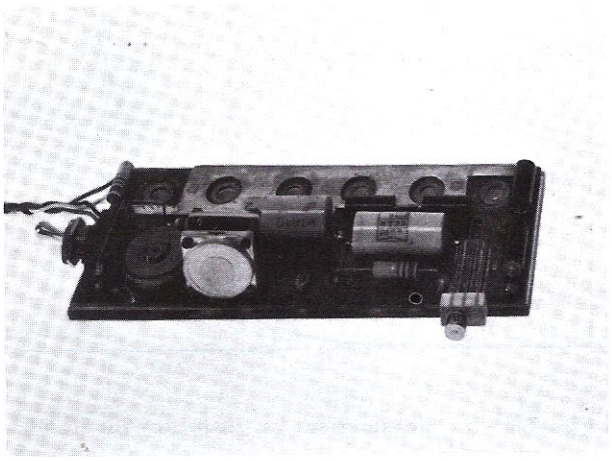


Fig. 22 - Close-up view of y_{fe} jig for low frequency

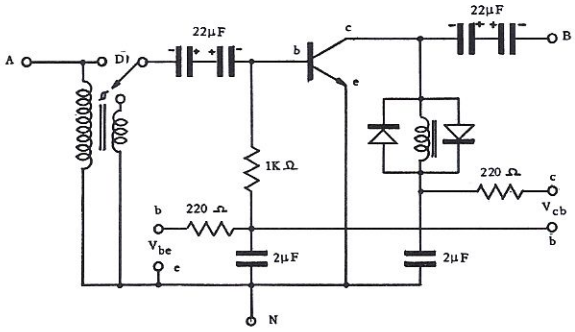


Fig. 20 - y_{fe} jig

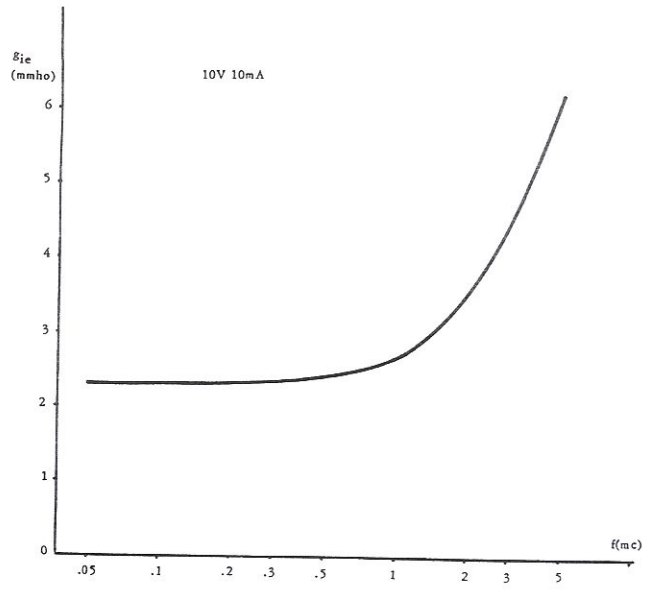


Fig. 23a - y_{ie} measurement vs. frequency for 2N2303

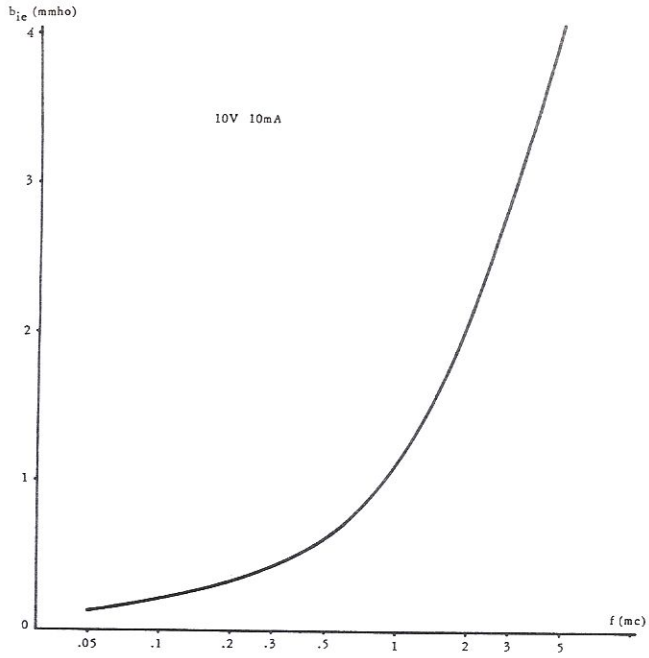


Fig. 23b - y_{ie} measurement vs. frequency for 2N2303

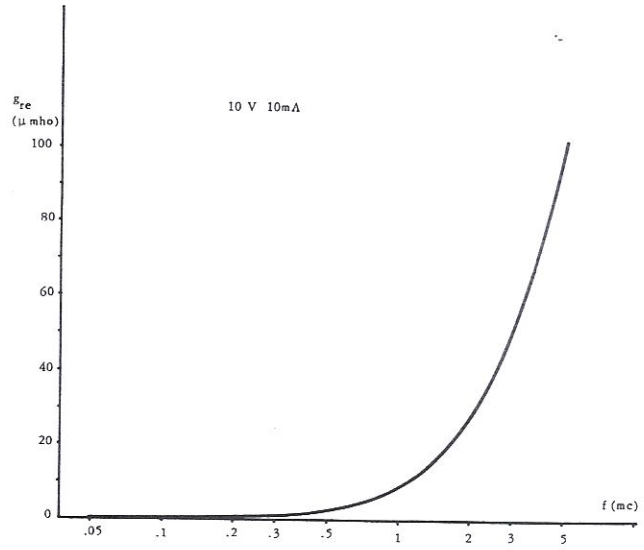


Fig. 24a - y_{re} measurement vs. frequency for 2N2303

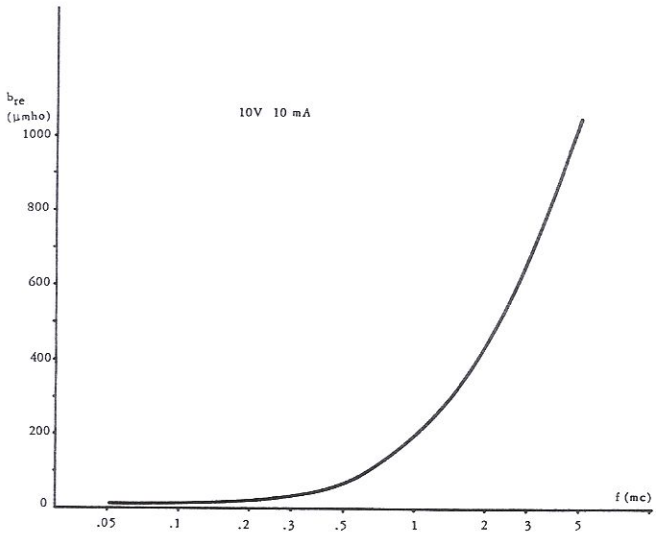


Fig. 24b - y_{re} measurement vs. frequency for 2N2303

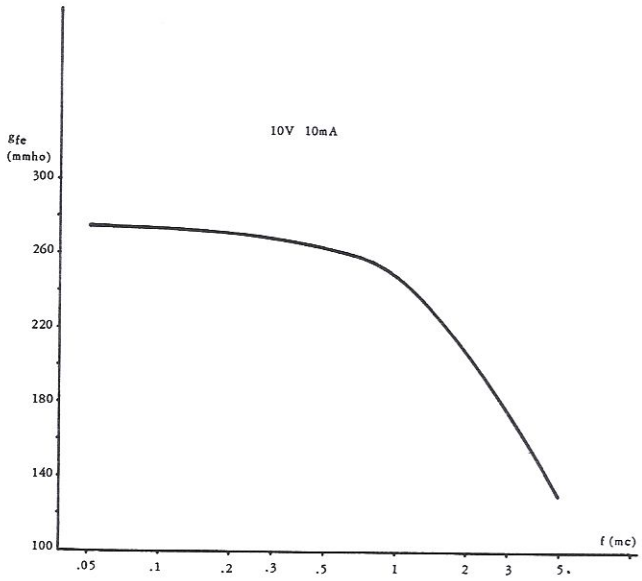


Fig. 25a - y_{fe} measurement vs. frequency for 2N2303

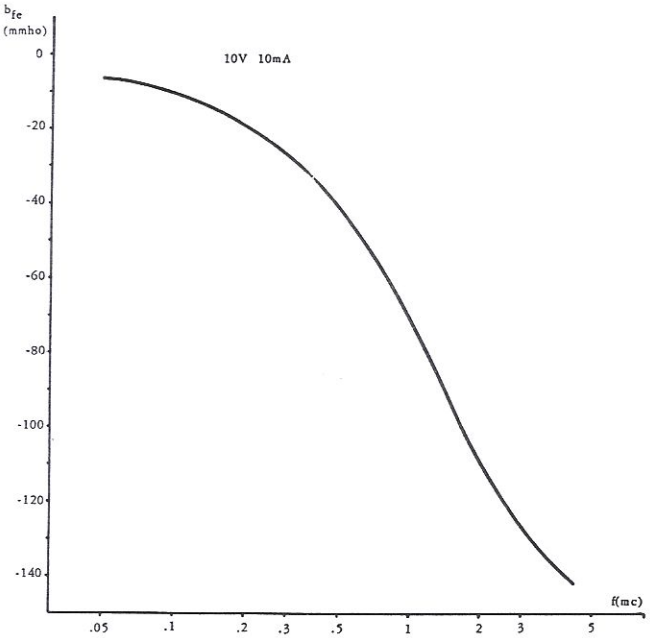


Fig. 25b - y_{fe} measurement vs. frequency for 2N2303

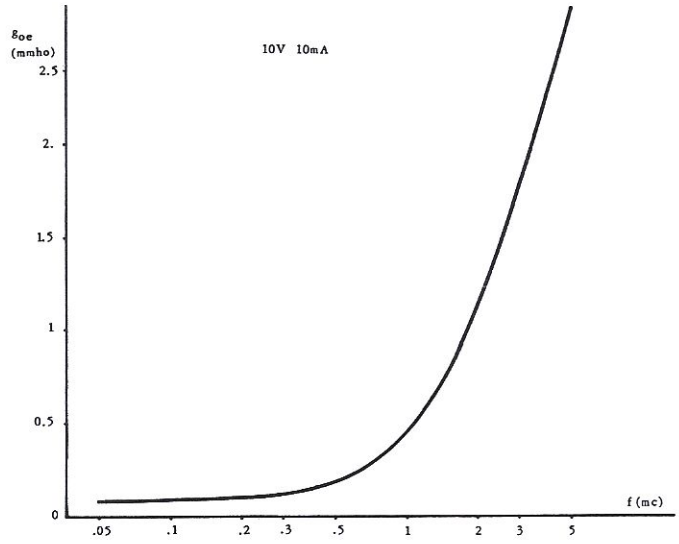


Fig. 26a - y_{oe} measurement vs. frequency for 2N2303

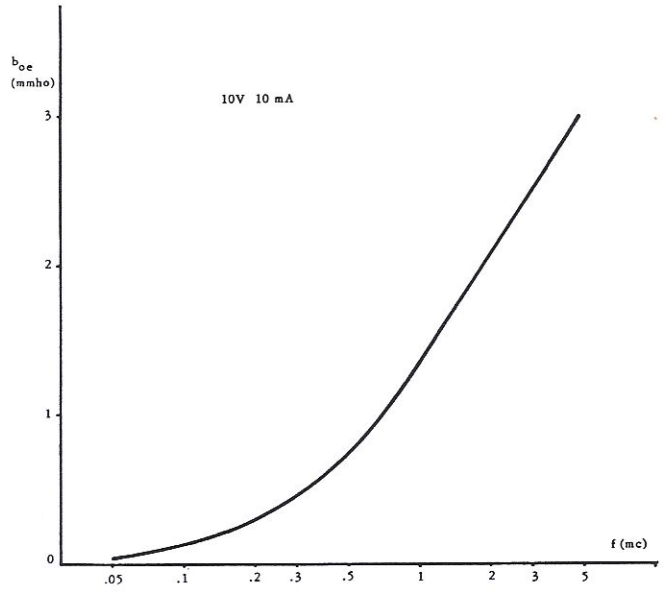


Fig. 26b - y_{oe} measurement vs. frequency for 2N2303

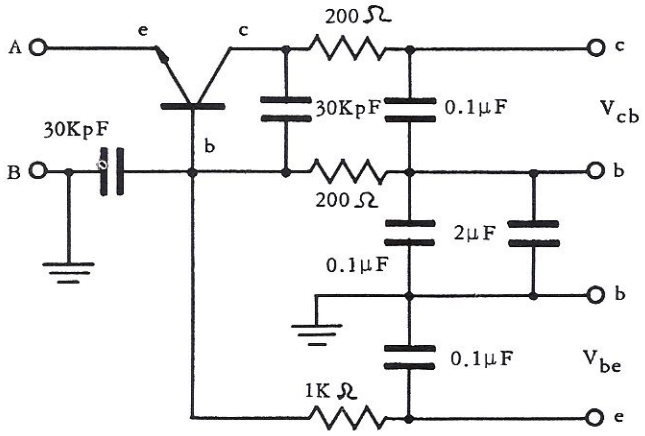


Fig. 27 - y_{ib} jig

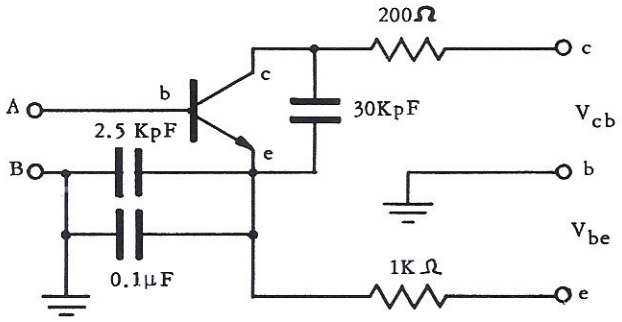


Fig. 28 - y_{ie} jig

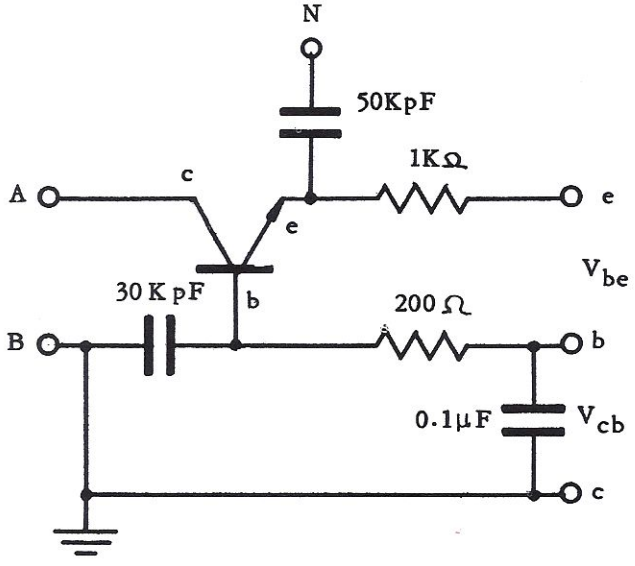


Fig. 29 - y_{re} jig

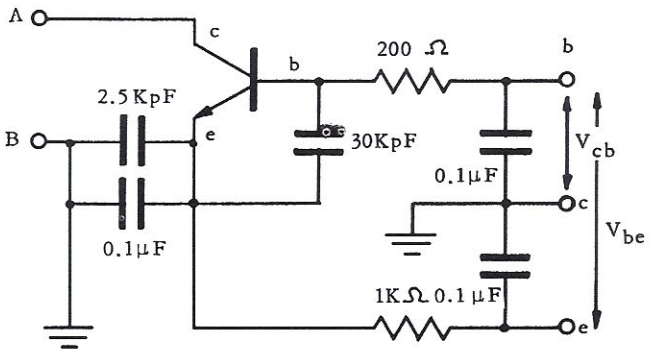


Fig. 30 - y_{oe} jig

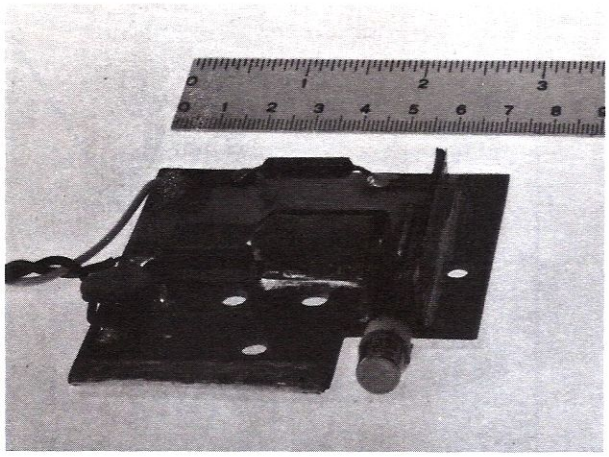


Fig. 31 - Close up view of y_{re} jig for high frequency

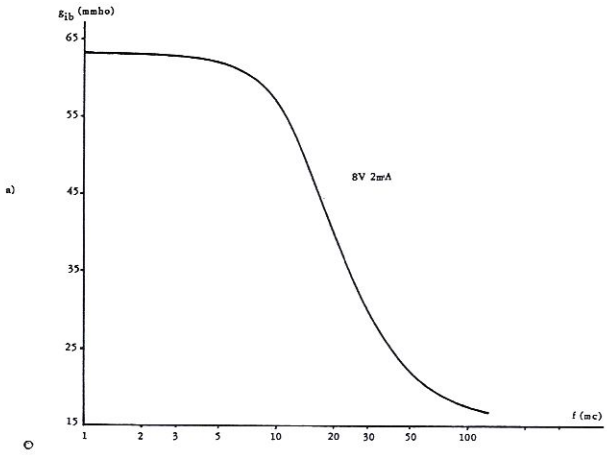


Fig. 32a - y_{ib} measurement vs. frequency for 2N916

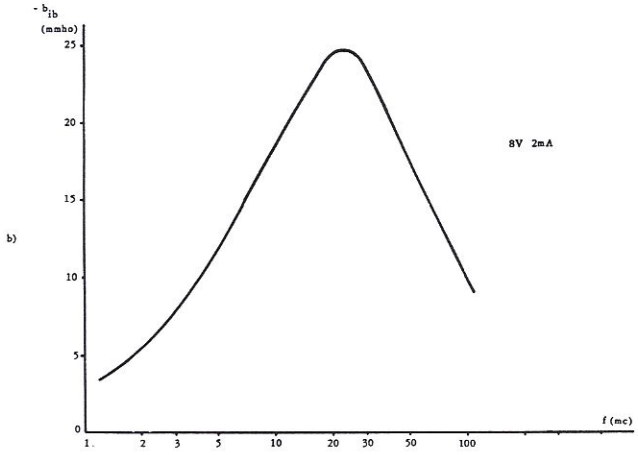


Fig. 32b - y_{ib} measurement vs. frequency for 2N916

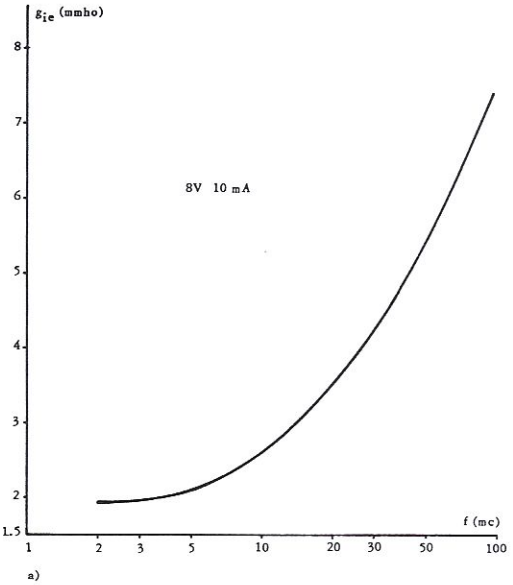


Fig. 33a - y_{ie} measurement vs. frequency for 2N916

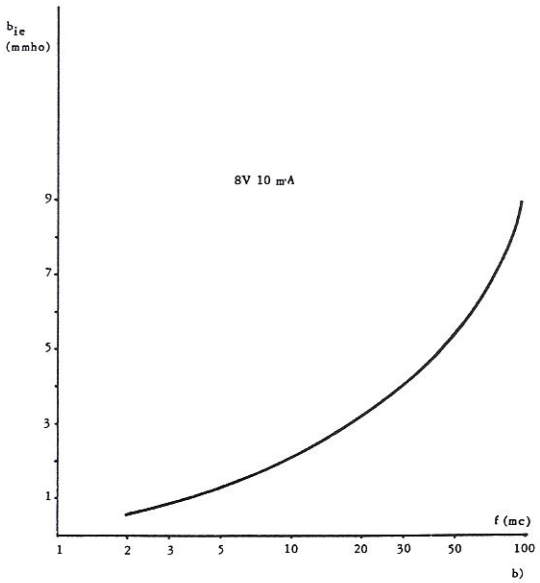


Fig. 33b - y_{ie} measurement vs. frequency for 2N916

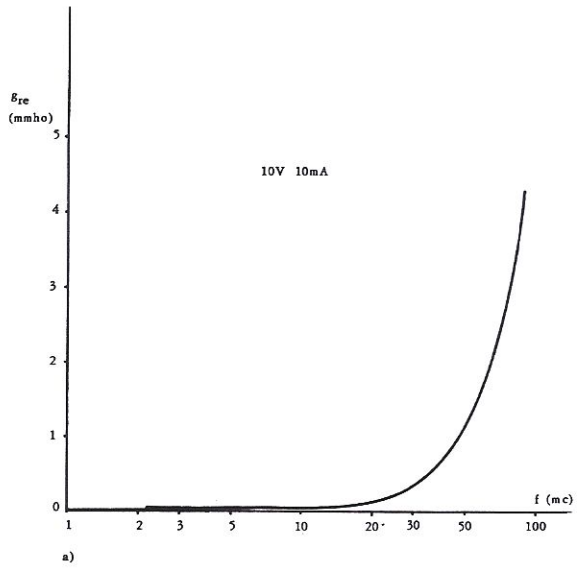


Fig. 34a - y_{re} measurement vs. frequency for 2N1613

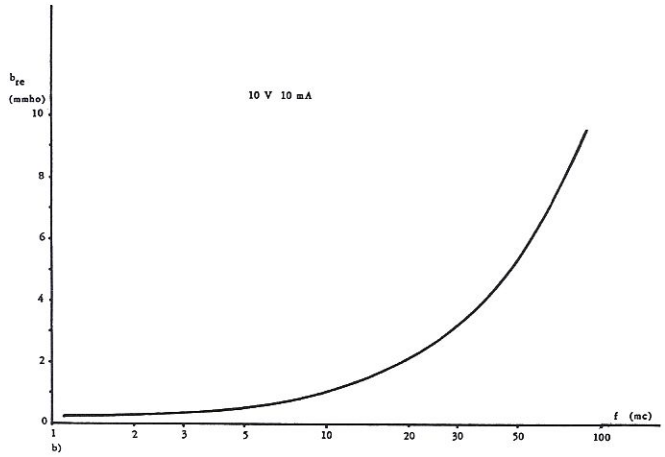


Fig. 34b - y_{re} measurement vs. frequency for 2N1613

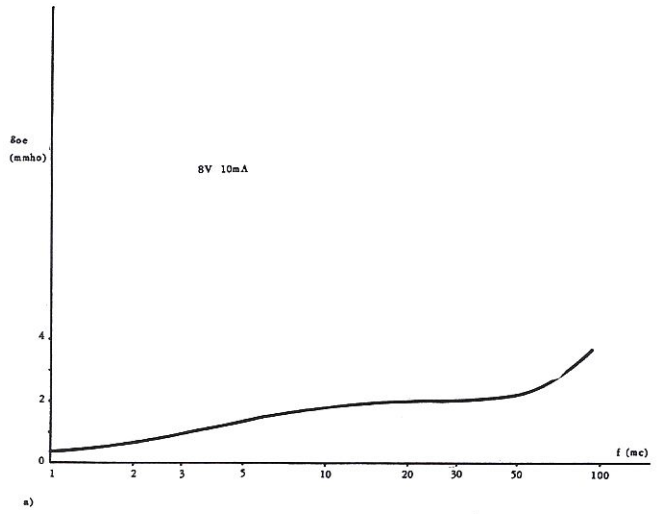


Fig. 35a - y_{oe} measurement vs. frequency for 2N1613

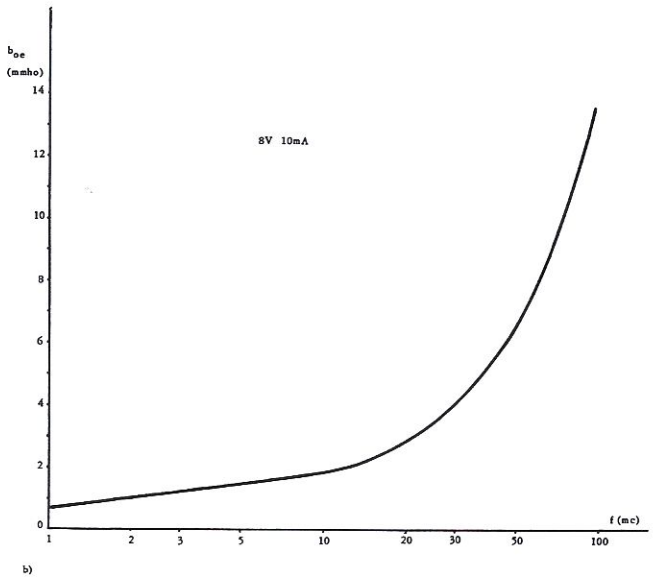


Fig. 35b - y_{oe} measurement vs. frequency for 2N1613